Reply to "Comment on 'Absence of chaos in a self-organized critical coupled map lattice' "

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(Received 16 March 1995)

We have compared our results with the reports pointed out to us by Ruxton (Phys. Rev. E, preceding Comment), but we have not found crucial contradictions. As a consequence of spatial dispersity, our model also shows local chaos in an enhanced parameter range, but global chaos is not present, similarly to the other observations.

PACS number(s): 05.40.+j, 05.45.+b, 05.70.Jk

In his comment [1], Ruxton compares our results [2] with a series of related publications [3] from the ecological literature. He finds a seeming contradiction: We did not observe low-dimensional chaos in our model, although other simulations suggest that in spatially distributed populations, chaos is more likely. Here we intend to show that there is no real contradiction between our results and earlier reports, if one carefully compares the statements belonging together.

One specific example cited extensively by Ruxton is the model investigated by Bascompte and Solé (BS) [4]. Indeed, BS chose the same map for local dynamics as we chose. However, while BS modeled the interaction between neighboring local populations via normal diffusion [4], we introduced a threshold condition and a time-scale separation in the spatial migration step [2]. Time-scale separation means that dispersal events are considered to be much faster than reproduction. Certainly, this assumption fails to concern bacteria or mammals, but flying insects of high mobility and of slow reproduction rate can be good candidates for comparison. [For example, the cockchafer (*Polyphylla fullo*) has a reproduction cycle of 4–5 years, but it can migrate several kilometers in a few days.] In spite of the fact that fast diffusional interaction triggered by a threshold condition may result in fundamentally different behavior than that of normal diffusion, we found that there is no sharp difference between BS's and our observation.

Let us repeat our main conclusion [2]: There is no sign of low-dimensional *collective* chaos in a system of local populations diffusively coupled to the neighboring ones and obeying a threshold condition. Emphasis is on *collective dynamics*, which intends to mimic the behavior of a metapopulation. Why have we concentrated on global behavior? To answer this we borrow BS's formulation: "As the bulk of the data record involves population census over a given area, they would be closer to global dynamics than local ones." BS did not systematically analyze the global behavior of their model, however, it seems that they did not find low-dimensional global chaos, either. This is not surprising, because it is more or less well established that noisless coupled map lattices with nearest neighbor diffusive coupling do not exhibit collective chaos [5]. The lack of collective chaos in our model is not as clear, because, as we explained [2], the time-scale separation may build up *long range effective correlations* in the system.

Concerning the *local dynamics*, BS found that increasing lattice size, as well as increasing the diffusional con-

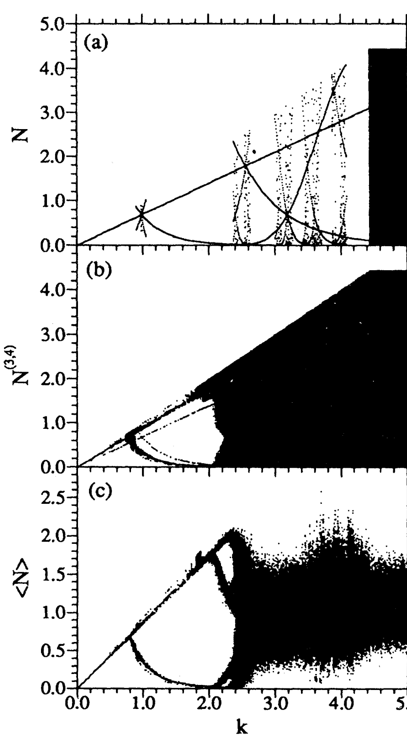

FIG. 1. Bifurcation diagrams as a function of the threshold parameter k, (a) a single map with open boundaries, (b) the site $i = 3$, $j = 4$ in a lattice of size $L = 8$, and (c) the lattice average in the same system as (b). See text.

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stant, extends the range of *local* chaos to a parameter range, where chaos is not present in an isolated map [4]. To check the role of spatial dispersity in our model, we performed a bifurcation analysis similarly to BS. In Fig. 1 we plotted the values of 1000 iterations after discarding 1000 initial points for different thresholds, keeping the other parameters at the strong chaotic range ($\lambda = 100.0$, $a = 1.0$, $\beta = 8.8$, see Ref. [2] for notations). Figure 1(a) refers to a single map with open boundaries: When the populational density exceeds the threshold k, at the next step the value will be $N_{sc} = 0.7k$ as a consequence of outward migrations. The threshold rule destroys chaos immediately after becoming effective ($k < 4.4304$). This compares with Fig. 1(b), where the appropriate values for a single site in a lattice of $L = 8 \times 8$ are plotted. In this spatially extended system, the local evolution remains chaotic (or at least complex) in a wide threshold range ($k > 2.350$), where a single map shows strict periodicity. This result is in agreement with BS's [4] and others observations [3], i.e., *local* chaos is more likely in distributed systems. However, we think that there is no way to observe such a local chaos in a natural metapopulation. One reason is that a local habitat in nature is not as well defined a unit as a site in a lattice model, the area, location, borders, etc., are changing dynamically in time, if it is possible to define them at all.

Figure 1(c) shows the asymptotic values for the whole lattice. At large threshold values ($k > 2.65$), the behavior is noiselike in the sense that the lattice average fluctuates around an (temporal) average with a Gaussian amplitude distribution, and the width of this Gaussian decreases according to the well-known square-root law with increasing number of lattice sites. Note that similar noisy behavior was observed by BS, too (cf. Fig. 7 in Ref. [4]), for the global dynamics. At smaller threshold values, period-four, period-two, and fixpoint behavior settle down, but *low-dimensional collective chaos* has not been found. Obviously, the noiselike behavior is not "real noise," because the fluctuations in the average are the consequences of deterministic rules, there is no external source of perturbations. This also means that the dynamics may obey "high-dimensional chaos," however, from the point of view of practical evaluation of field observations, this makes no real difference. Low-dimensional collective chaos would mean that there exists one or at most a few "master" degrees of freedom governing the time evolution of an otherwise complex system, and obeying some nonlinear equation(s) of motion.

What are the main differences now between BS's and our model? One is very important: While the diffusive coupling *without* a threshold condition and time-scale separation makes the dispersed system *inherently instable*, therefore, an additional rule was necessary for BS to avoid negative values, our model does not need such a (populational dynamically unreasonable) adjusting step. Moreover, it is interesting that we did not find spatially ordered patterns when the system size is large enough and the local dynamics is strongly chaotic, which shows that external noise is not necessary when mimicing more a natural spatial distribution of populations.

Finally, let us comment on childhood diseases as a good example for natural chaos [1]. We share the opinion of Ruxton that these are a time series of "best quality" to check different models from the point of view of chaos as well. In spatially dispersed disease models, however, one should consider that human mobility is far from being a nearest neighbor interaction, and a nonlocal connection may lead synchronized chaos even at surprisingly low coordination numbers [6].

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